

Black-hole horizon and metric singularity at the brane separating two sliding superfluids

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Abstract

An analog of black hole can be realized in the low- temperature laboratory. The horizon can be constructed for the ‘relativistic’ ripplons (surface waves) living on the brane. The brane is represented by the interface between two superfluid liquids, $^3\text{He-A}$ and $^3\text{He-B}$, sliding along each other without friction. Similar experimental arrangement has been recently used for the observation and investigation of the Kelvin-Helmholtz type of instability in superfluids [1]. The shear-flow instability in superfluids is characterized by two critical velocities. The lowest threshold measured in recent experiments [1] corresponds to appearance of the ergoregion for ripplons. In the modified geometry this will give rise to the black-hole event horizon in the effective metric experienced by ripplons. In the region behind the horizon, the brane vacuum is unstable due to interaction with the higher-dimensional world of bulk superfluids. The time of the development of instability can be made very long at low temperature. This will allow us to reach and investigate the second critical velocity – the proper Kelvin-Helmholtz instability threshold. The latter corresponds to the singularity inside

the black hole, where the determinant of the effective metric becomes infinite.

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1. Introduction. The first experimental realization of two superfluid liquids sliding along each other [1] gives a new tool for investigation of many physical phenomena related to different areas of physics (classical hydrodynamics, rotating Bose condensates, cosmology, brane physics, etc.). Here we discuss how this experimental arrangement can be modified in order to produce an analog of the black-hole event horizon and of the singularity in the effective Lorentzian metric experienced by the collective modes (ripplons) living on the brane (the interface separating two different superfluid vacua, ${}^3\text{He-A}$ and ${}^3\text{He-B}$, which we further refer as the AB-brane).

The idea of the experiment is similar to that discussed by Schützhold and Unruh [2], who suggested to use the gravity waves on the surface of a liquid flowing in a shallow basin. In the long-wavelength limit the energy spectrum of the surface modes becomes ‘relativistic’, which allows us to describe the propagating modes in terms of the effective Lorentzian metric. Here we discuss the modification of this idea to the case of the ripplons propagating along the membrane between two superfluids.

There are many advantages when one uses the superfluid liquids instead of the conventional ones: (1) The superfluids can slide along each other without any friction until the critical velocity is reached, and thus all the problems related to viscosity disappear. (2) The superfluids represent the quantum vacua similar to that in relativistic quantum field theories (RQFT) (see review [3]). That is why the quantum effects related to the vacuum in the presence of exotic metric can be simulated. (3) The interface between two different superfluid vacua is analogous to the brane in the modern RQFT, and one can study the brane physics, in particular the interaction between the brane matter and the matter living in the higher-dimensional space outside the brane. Here, on example of the AB-brane, we show that this interaction leads to the vacuum instability in the AB-brane behind the event horizon. (4) Reducing the temperature one can make the time of the development of the instability long enough to experimentally probe the singularity within the black hole (the so-called physical singularity).

2. Effective metric for modes living in the AB-brane. Let us consider surface waves – ripplons – propagating along the AB-brane in the slab geometry shown in Fig.1.

Two superfluids, ${}^3\text{He-A}$ and ${}^3\text{He-B}$, separated by the AB-brane are moving along the brane with velocities \mathbf{v}_1 and \mathbf{v}_2 in the container

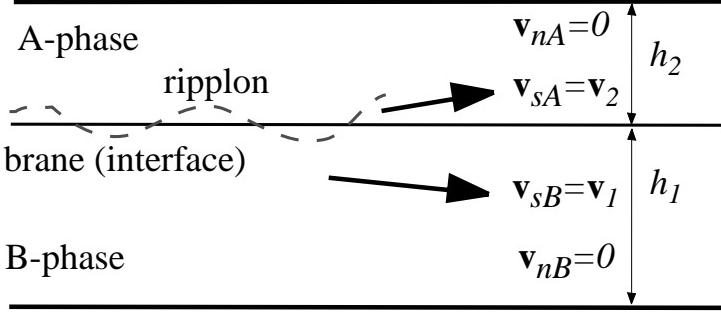


Figure 1: The brane – interface between two moving superfluids, ${}^3\text{He-A}$ and ${}^3\text{He-B}$. \mathbf{v}_{sA} and \mathbf{v}_{sB} are the superfluid velocities of two liquids sliding along the brane, while the normal components of the liquids – analog of the matter living outside the brane – are at rest in the frame of container, $\mathbf{v}_{nA} = \mathbf{v}_{nB} = 0$. The dashed line demonstrates the propagating surface wave (ripplon) which represents the matter living on the brane

frame. The normal components of the liquids – the systems of quasi-particles on both sides of the interface – are at rest with respect of the container walls in equilibrium, $\mathbf{v}_n = 0$. The dispersion relation for ripplons can be obtained by modification of the equations obtained in Ref. [4] to the slab geometry:

$$\begin{aligned} M_1(k)(\omega - \mathbf{k} \cdot \mathbf{v}_1)^2 + M_2(k)(\omega - \mathbf{k} \cdot \mathbf{v}_2)^2 &= \\ &= F + k^2\sigma - i\Gamma\omega. \end{aligned} \quad (1)$$

Here σ is the surface tension of the AB-brane; F is the force stabilizing the position of the brane, in experiment [1] it is an applied magnetic-field gradient; $M_1(k)$ and $M_2(k)$ are masses of the two liquids involved into the oscillating motion of the brane:

$$M_1(k) = \frac{\rho_1}{k \tanh kh_1}, \quad M_2(k) = \frac{\rho_2}{k \tanh kh_2}; \quad (2)$$

h_1 and h_2 are thicknesses of layers of two superfluids; ρ_1 and ρ_2 are mass densities of the liquids, we assume that the temperature is low enough so that the normal fraction of each of two superfluid liquids is small.

Finally Γ is the coefficient in front of the friction force experienced by the AB-brane when it moves with respect to the 3D environment along the normal $\hat{\mathbf{z}}$ to the brane, $\mathbf{F}_{\text{fr}} = -\Gamma(\mathbf{v}_{\text{brane}} - \mathbf{v}_n)$ (in the frame

of container $\mathbf{v}_n = 0$). The friction term in Eq.(1) containing the parameter Γ is the only term which couples the 2D brane with the 3D environment. If $\Gamma = 0$, the brane subsystem becomes Galilean invariant; the Γ -term violates Galilean invariance in the 2D world of the AB-brane.

In a thin slab where $kh_1 \ll 1$ and $kh_2 \ll 1$ one obtains

$$\begin{aligned} \alpha_1(\omega - \mathbf{k} \cdot \mathbf{v}_1)^2 + \alpha_2(\omega - \mathbf{k} \cdot \mathbf{v}_2)^2 &= \\ = c^2 k^2 \left(1 + \frac{k^2}{k_P^2}\right) - 2i\tilde{\Gamma}(k)\omega, \end{aligned} \quad (3)$$

where

$$\alpha_1 = \frac{h_2\rho_1}{h_2\rho_1 + h_1\rho_2}, \quad \alpha_2 = 1 - \alpha_1 = \frac{h_1\rho_2}{h_2\rho_1 + h_1\rho_2}, \quad (4)$$

$$k_P^2 = \frac{F}{\sigma}, \quad c^2 = \frac{Fh_1h_2}{h_2\rho_1 + h_1\rho_2}, \quad \tilde{\Gamma}(k) = \frac{\Gamma}{2}k^2 \frac{h_1h_2}{h_2\rho_1 + h_1\rho_2} \quad (5)$$

For $k \ll k_P$ the main part of Eq. (3) can be rewritten in the Lorentzian form

$$g^{\mu\nu} k_\mu k_\nu = 2i\omega\tilde{\Gamma}(k) - c^2 k^4/k_P^2, \quad (6)$$

$$k_\mu = (-\omega, k_x, k_y), \quad k = \sqrt{k_x^2 + k_y^2}, \quad (7)$$

while the right-hand side of Eq.(6) contains the remaining small terms violating Lorentz invariance – attenuation of ripplons due to the friction and their nonlinear dispersion. Both terms come from the physics which is ‘trans- Planckian’ for the ripplons. The quantities k_P and ck_P play the role of the Planck momentum and Planck energy within the brane: they determine the scales where the Lorentz symmetry is violated. The Planck scales of the 2D physics in brane are actually much smaller than the ‘Planck momentum’ and ‘Planck energy’ in the 3D superfluids outside the brane. The parameter Γ is determined by the physics of 3D quasiparticles scattering on the brane, and it practically does not depend on velocities \mathbf{v}_1 and \mathbf{v}_2 , which are too small for the 3D world.

At sufficiently small k both non-Lorentzian terms – attenuation and nonlinear dispersion on the right-hand side of Eq.(6) – can be ignored, and the dynamics of ripplons living on the AB-brane is described by the following effective contravariant metric $g^{\mu\nu}$:

$$\begin{aligned} g^{00} &= -1, \quad g^{0i} = -\alpha_1 v_1^i - \alpha_2 v_2^i, \\ g^{ij} &= c^2 \delta^{ij} - \alpha_1 v_1^i v_1^j - \alpha_2 v_2^i v_2^j. \end{aligned} \quad (8)$$

Introducing relative velocity \mathbf{U} and the mean velocity \mathbf{W} of two superfluids:

$$\mathbf{W} = \alpha_1 \mathbf{v}_1 + \alpha_2 \mathbf{v}_2, \quad \mathbf{U} = \mathbf{v}_1 - \mathbf{v}_2, \quad (9)$$

one obtains the following expression for the effective contravariant metric

$$\begin{aligned} g^{00} &= -1, \\ g^{0i} &= -W^i, \quad g^{ij} = c^2 \delta^{ij} - W^i W^j - \alpha_1 \alpha_2 U^i U^j. \end{aligned} \quad (10)$$

3. Horizon and singularity. The original KH instability [5] takes place when the relative velocity U of the motion of the two liquids reaches the critical value $U_c = c/\sqrt{\alpha_1 \alpha_2}$. At this velocity the determinant of the metric tensor

$$g(r) = -\frac{1}{c^2 (c^2 - \alpha_1 \alpha_2 U^2(r))}, \quad (11)$$

has a physical singularity: it crosses the infinite value and changes sign. However, before U reaches U_c , the system reaches the other important thresholds at which analogs of ergosurface and horizon in general relativity appear. To demonstrate this, let us consider the simplest situation when velocities \mathbf{U} and \mathbf{W} are parallel to each other (i.e. \mathbf{v}_1 and \mathbf{v}_2 are parallel); and these velocities are radial and depend only on the radial coordinate r along the flow. Then the interval of the effective 2+1 space-time in which ripplons move along the geodesic curves is

$$\begin{aligned} ds^2 &= \\ &= \frac{-(c^2 - W^2(r) - \alpha_1 \alpha_2 U^2(r)) dt^2 - 2W(r) dt dr + dr^2}{c^2 - \alpha_1 \alpha_2 U^2(r)} + \\ &\quad + r^2 d\phi^2 = \\ &= -d\tilde{t}^2 \frac{c^2 - W^2(r) - \alpha_1 \alpha_2 U^2(r)}{c^2 - \alpha_1 \alpha_2 U^2(r)} + \\ &\quad + \frac{dr^2}{c^2 - W^2(r) - \alpha_1 \alpha_2 U^2(r)} + r^2 d\phi^2, \end{aligned} \quad (13)$$

$$d\tilde{t} = dt + \frac{W(r) dr}{c^2 - W^2(r) - \alpha_1 \alpha_2 U^2(r)}. \quad (14)$$

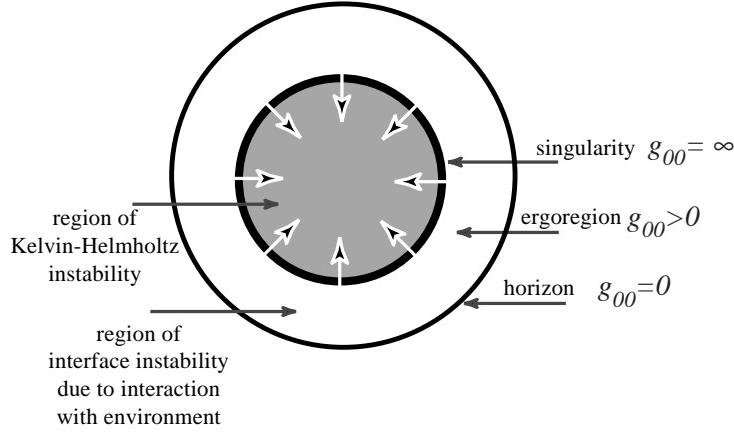


Figure 2: Horizon and singularity in the effective metric for ripplons on the brane (AB-interface). We assume that the A-phase is at rest, while the B-phase is radially moving to the center as shown by arrows.

The circle $r = r_h$, where $g_{00} = 0$, i.e. where $W^2(r_h) - \alpha_1 \alpha_2 U^2(r_h) = c^2$, marks the ‘coordinate singularity’ which is the black-hole horizon if the velocity W is inward (see Fig.2). In such radial-flow geometry the horizon also represents the ergosurface (ergoline in 2D space dimension) which is determined as the surface bounding the region where the ripplon states can have the negative energy. We call the whole region behind the ergosurface the ergoregion. This definition differ from that accepted in general relativity, but we must extend the notion of the ergoregion to the case when the Lorentz invariance and general covariance are violated, and the absolute reference frame appears. At the ergosurface, the Landau critical velocity for excitations of ripplons is reached. And also, as follows from Ref. [4] (see also section 5 below), the ergoregion coincides with the region where the brane fluctuations go unstable, since both real and imaginary (Fig.3) parts of the ripplon energy spectrum cross zero at the ergosurface.

4. Brane instability behind horizon. This means that the brane becomes unstable in the presence of the ergoregion. This instability is caused by the interaction of the 2D ripplons with the 3D quasiparticles living in bulk superfluids on both sides of the brane [4]. The interaction of brane with the environment, i.e. with the superfluids on both sides of the brane, is the source of the attenuation of

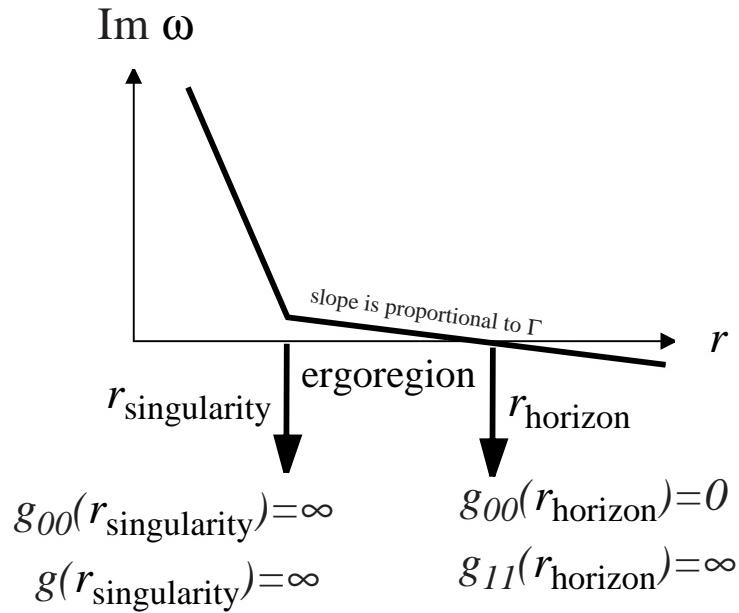


Figure 3: Imaginary part of the ripplon spectrum due to interaction with the environment in the higher-dimensional space. In the ergoregion the attenuation transforms to the amplification leading to the instability of the brane world. The time of development of this instability is long at low T , where Γ is small. On the contrary, the Kelvin-Helmholtz instability behind the singularity rapidly develops and $r_{\text{singularity}} \rightarrow 0$.

the propagating ripplons: this interaction determines the parameter Γ in the friction force. In the ergoregion, the imaginary part of the spectrum of ripplons becomes positive, i.e. the attenuation transforms to amplification of surface waves with negative ω (see Figure 3 where the imaginary part of the spectrum crosses zero with the slope proportional to Γ). Since the instability of the interface with respect to exponentially growing surface fluctuations develops in the presence of the shear flow, this instability results in the formation of vortices observed in experiment [1].

In ${}^3\text{He}$ experiments [1] with shear flow along the AB-interface, one has $kh_1 \gg 1$ and $kh_2 \gg 1$. Thus the relativistic description is not applicable. Also, in the rotating cryostat the superfluids flow in the azimuthal direction instead of the radial. That is why there was no horizon in the experiment. However, the notion of the ergosurface and of the ergoregion behind the ergosurface, where the ripplon energy becomes negative in the container frame [4], is applicable. The instability of the brane inside the ergoregion leads to formation of vortices in the vortex-free ${}^3\text{He-B}$, which were detected using NMR technique with a single vortex resolution. The observed threshold velocity for the vortex formation exactly corresponds to the appearance of the ergosurface (ergoline) in the container [1, 4].

There are thus two ingredients which cause the vacuum instability in the ergoregion: (i) existence of the absolute reference frame of the environment outside the brane; (ii) the interaction of the brane with this environment ($\Gamma \neq 0$) which violates Galilean (or Lorentz) invariance within the brane. They lead to attenuation of the ripplon in the region outside the horizon. Behind the horizon this attenuation transforms to amplification which destabilizes the vacuum there. This mechanism may have an important consequence for the astronomical black hole. If there is any intrinsic attenuation of, say, photons (either due to superluminal dispersion, or due to the interaction with the higher-dimensional environment), this may lead to the catastrophic decay of the black hole due to instability behind the horizon, which we discuss in the Section 5.

Let us estimate the time of development of such instability, first in the artificial black hole within the AB-brane and then in the astronomical black hole. According to Kopnin [6] the parameter of the friction force experienced by the AB-brane due to Andreev scattering of quasiparticles living in the bulk superfluid on the A-phase side of

the brane is $\Gamma \sim T^3 m^* / \hbar^3 c_\perp c_\parallel$ at $T \ll T_c$. Here T is the temperature in ${}^3\text{He-A}$; m^* is the quasiparticle mass in the Fermi liquid; c_\perp and c_\parallel are the ‘speeds of light’ for 3D quasiparticles living in anisotropic ${}^3\text{He-A}$ (these speeds are much larger than the typical ‘speed of light’ c of quasiparticles (ripplons) living on 2D brane); T_c is the superfluid transition temperature, which also marks the 3D Planck energy scale. Assuming the most pessimistic scenario in which the instability is caused mainly by the exponential growth of ripplons with the ‘Planck’ wave number k_P , one obtains the following estimation for the time of the development of the instability in the ergoregion far enough from the horizon: $\tau \sim 1/\tilde{\Gamma}(k_P) \sim 10(T_c/T)^3$ sec. Thus at low T the state with the horizon can live for a long time (minutes or even hours), and this life-time of the horizon can be made even longer if the threshold is only slightly exceeded.

This gives the unique possibility to study the horizon, the region behind the horizon; and the physical singularity, where the determinant of the metric is singular, can be also easily constructed and investigated.

At lower temperature $T < m^* c_\perp^2$ the temperature dependence of Γ changes: $\Gamma \sim T^4 / \hbar^3 c_\perp^3 c_\parallel$ [3], and at very low T it becomes temperature independent: $\Gamma \sim \hbar k^4$ which corresponds to the dynamical Casimir force acting on the 2D brane moving in the 3D vacuum. Such intrinsic attenuation of ripplons transforms to the amplification of the ripplon modes in the ergoregion, which leads to instability of the brane vacuum behind the horizon even at $T = 0$.

5. Instability of the black hole behind horizon? Now let us suppose that the same situation takes place in our (brane) world, i.e. the modes of our world (photons, or gravitons, or fermionic particles) have finite life-time due to interaction with, say, the extra-dimensional environment. Then this will lead to the instability of vacuum behind the horizon of the astronomical black holes. This can be considered using the equation (6) which incorporates both terms violating the Lorentz invariance at high energy: the superluminal upturn of the spectrum which leads to decay of particles, and the intrinsic broadening of the particle spectrum characterized by $\tilde{\Gamma}(k)$. Following the analogy, we can write the intrinsic width as a power law $\tilde{\Gamma}(k) \sim \mu(ck/\mu)^n$, where μ is the energy scale which is well above the Planck scale E_P of our brane world, $\mu \gg E_P$; and $n = 6$ if the analogy is exact.

We shall use the Painlevé-Gullstrand metric, which together with

the superluminal dispersion of the particle spectrum allows us to consider the region behind the horizon:

$$\begin{aligned} g^{00} &= -1, \quad g^{0i} = -W^i, \\ g^{ij} &= c^2 \delta^{ij} - W^i W^j, \quad \mathbf{W} = -\hat{\mathbf{r}} \sqrt{\frac{2GM}{r}}. \end{aligned} \quad (15)$$

Here G is Newton constant and M is the mass of the black hole. This metric coincides with the 3D generalization of the metric of ripplons on AB-brane in Eq. (13) in case when $\mathbf{v}_1 = \mathbf{v}_2$. Equation (13) gives the following dispersion relation for particles living in the brane:

$$(\omega - \mathbf{k} \cdot \mathbf{W})^2 = c^2 k^2 - 2i\omega \tilde{\Gamma}(k) + c^2 k^4/k_P^2, \quad (16)$$

or

$$\begin{aligned} \omega(\mathbf{k}) &= \mathbf{k} \cdot \mathbf{W} - i\tilde{\Gamma}(k) \pm \\ &\pm \sqrt{c^2 k^2 + c^2 k^4/k_0^2 - \tilde{\Gamma}^2(k) - 2i\mathbf{k} \cdot \mathbf{W}\tilde{\Gamma}(k)}. \end{aligned} \quad (17)$$

We are interested in the imaginary part of the spectrum. For small $\tilde{\Gamma}(k) \ll ck$, the imaginary part of the energy spectrum is:

$$\begin{aligned} \text{Im } \omega(\mathbf{k}) &= -i\tilde{\Gamma}(k) \left(1 \pm \frac{\mathbf{k} \cdot \mathbf{W}}{E(k)} \right), \\ E^2(k) &= c^2 k^2 + c^2 k^4/k_P^2. \end{aligned} \quad (18)$$

Behind the horizon, where $W > c$, the imaginary part becomes positive for $|\mathbf{k} \cdot \mathbf{W}| > E(k)$ (or $k^2 < k_P^2/(W^2/c^2 - 1)$), i.e. attenuation transforms to amplification of waves with these \mathbf{k} . This demonstrates the instability of the vacuum with respect to exponentially growing electromagnetic or other fluctuations in the ergoregion. Such an instability is absent when $\tilde{\Gamma} = 0$, i.e. if there is no interaction with the trans-Planckian or extra-dimensional world(s).

The time of the development of instability within the conventional black hole is determined by the region far from the horizon, where the relevant $k \sim k_P$. Thus $\tau \sim 1/\tilde{\Gamma}(k_P) \sim \mu^{n-1}/E_P^n$. If μ is of the same order as the brane Planck scale, the time of development of instability is determined by the Planck time. That is why the astronomical black hole can exist only if $\mu \gg E_P$, which takes place when the 3D and 2D Planck scales are essentially different, as it happens in case of the AB-brane. The black hole decay due to the quantum process of Hawking evaporation corresponds to $\mu = M$, where M is a black hole mass, and $n = 4$.

6. Conclusion. In conclusion, the AB-brane – the interface between the two sliding superfluids – can be used to construct the artificial black hole with the ergosurface, horizon and physical singularity. Using the AB-brane one can also simulate the interaction of particles living on the brane with that living in the higher-dimensional space outside the brane. This interaction leads to the decay of the brane vacuum in the region behind the horizon. This mechanism can be crucial for the astronomical black holes, if this analogy is applicable. If the matter fields in the brane are properly coupled to, say, gravitons in the bulk, this may lead to the fast collapse of the black hole.

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